Foundations of Artificial Intelligence

Alberto Maria Metelli

Exercise Session
10-29-2021

Exercise 5.3

Solve the 4-Queens problem. The problem consists of placing 4 queens on a 4x4 chess board so that no queen can attack any other.

1. Formulate the problem as a constraint satisfaction problem
2. Solve it using backtracking with minimum-remaining-values heuristic and forward checking. Only one solution is required.

Exercise 5.3

$X = \{x_1, x_2, x_3, x_4\}$

x_i is the row at which the queen in column i is placed

$D = \{D_1, D_2, D_3, D_4\}$ $\quad$ $D_1 = D_2 = D_3 = D_4 = \{1, 2, 3, 4\}$

We can express the constraints in a compact form, with $i \in \{1, 2, 3, 4\}$ and $j \in \{1, 2, 3\}$:

$$C(x_i, x_{i+j}) = \{(a, b) : a,b \in \{1, 2, 3, 4\} , |a - b| \notin \{0, j\} \}$$

Exercise 5.3

In an extensive way:

$$C(X_1, X_2) = \{\langle 1,3\rangle, \langle 1,4\rangle, \langle 2,4\rangle, \langle 3,1\rangle, \langle 4,1\rangle, \langle 4,2\rangle\}$$

$$C(X_1, X_3) = \{\langle 1,2\rangle, \langle 1,4\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,1\rangle, \langle 4,3\rangle\}$$

$$C(X_1, X_4) = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 3,1\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,2\rangle, \langle 4,3\rangle\}$$

$$C(X_2, X_3) = C(X_1, X_2)$$

$$C(X_2, X_4) = C(X_1, X_3)$$

$$C(X_3, X_4) = C(X_1, X_2)$$

Exercise 5.3

Even if not requested by the exercise, we try to apply AC-3:

$Q = \{x_1 \to x_2, x_2 \to x_1, x_1 \to x_3, x_3 \to x_1, x_1 \to x_4, x_4 \to x_1, x_2 \to x_3, x_3 \to x_2, x_2 \to x_4, x_4 \to x_2, x_3 \to x_4, x_4 \to x_3\}$

$x_1 \to x_2$: nothing

$x_2 \to x_1$: nothing

$x_1 \to x_3$: nothing

Exercise 5.3

$x_3 \rightarrow x_1$: nothing

$x_1 \rightarrow x_4$: nothing

$x_4 \rightarrow x_1$: nothing

$x_2 \rightarrow x_3$: nothing

$x_3 \rightarrow x_2$: nothing

$x_2 \rightarrow x_4$: nothing

$x_4 \rightarrow x_2$: nothing

$x_3 \rightarrow x_4$: nothing

$x_4 \rightarrow x_3$: nothing

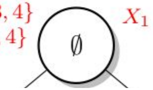
In this problem, AC-3 is unable to shrink the domains, but not all assignments of domain values is a solution!

Exercise 5.3

We apply **backtracking** with:

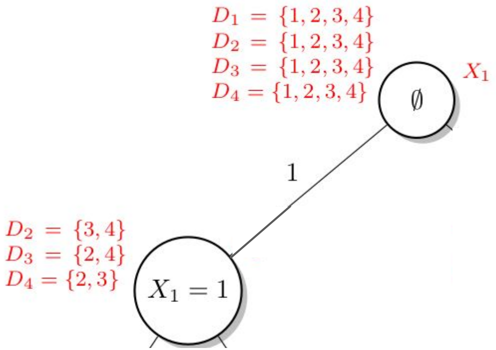
- minimum-remaining-values heuristic (**MRV**)
- forward checking (**FC**)

$$
\begin{aligned}
D_1 &= \{1,2,3,4\}\\
D_2 &= \{1,2,3,4\}\\
D_3 &= \{1,2,3,4\}\\
D_4 &= \{1,2,3,4\}
\end{aligned}
$$

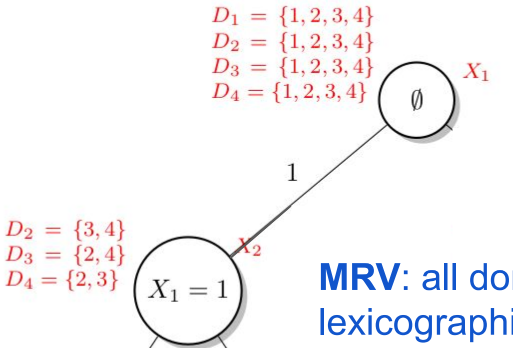


MRV: all domains have 4 values
→ lexicographical order

FC

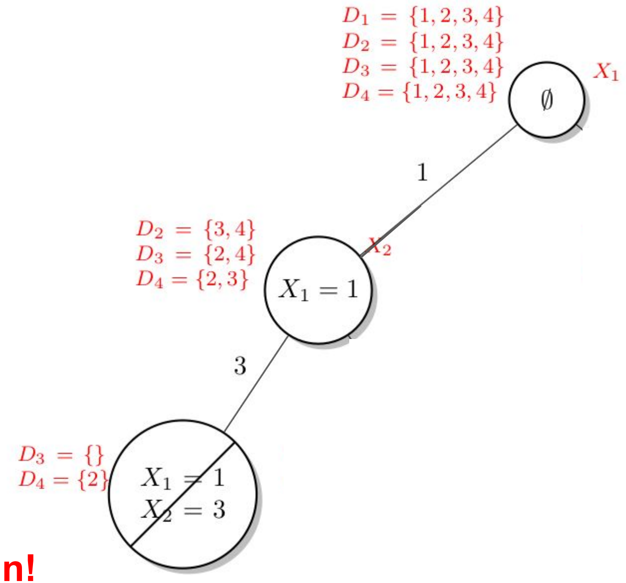


$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$

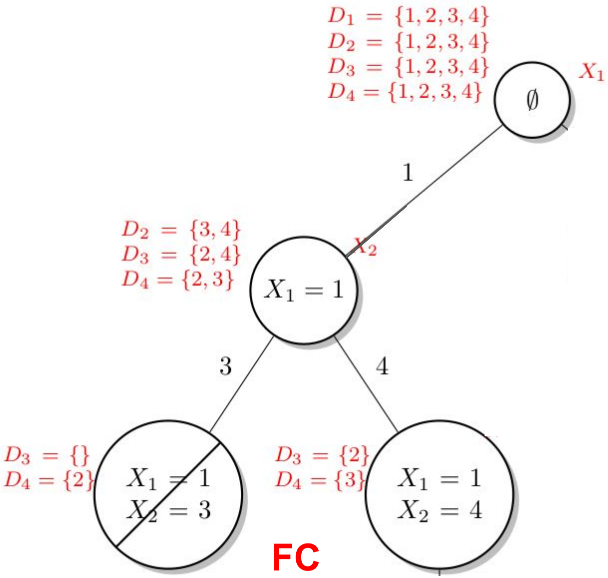


MRV: all domains have 4 values → lexicographical order

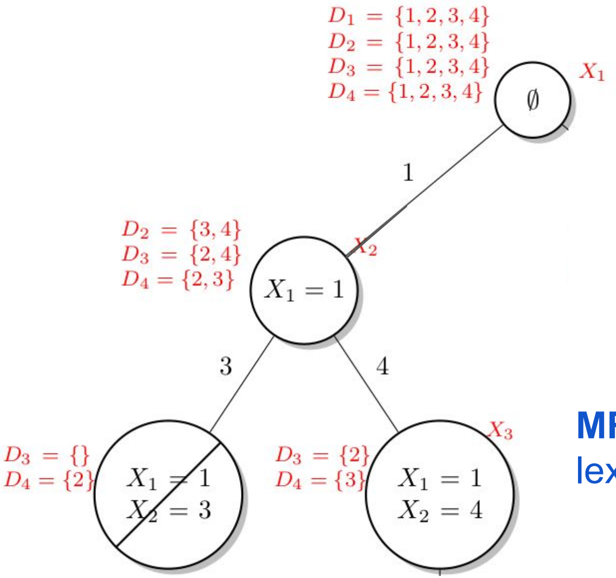
FC



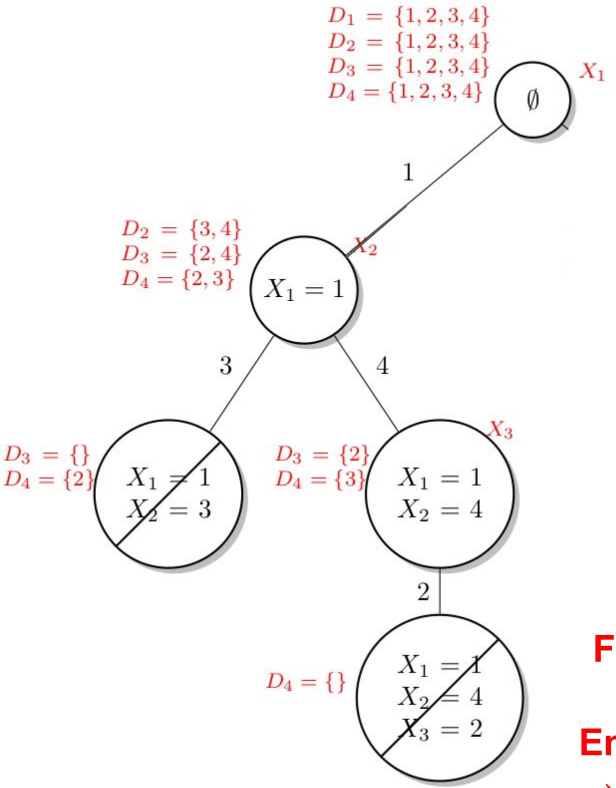
Empty domain!
→ No solution


$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
3
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$

FC

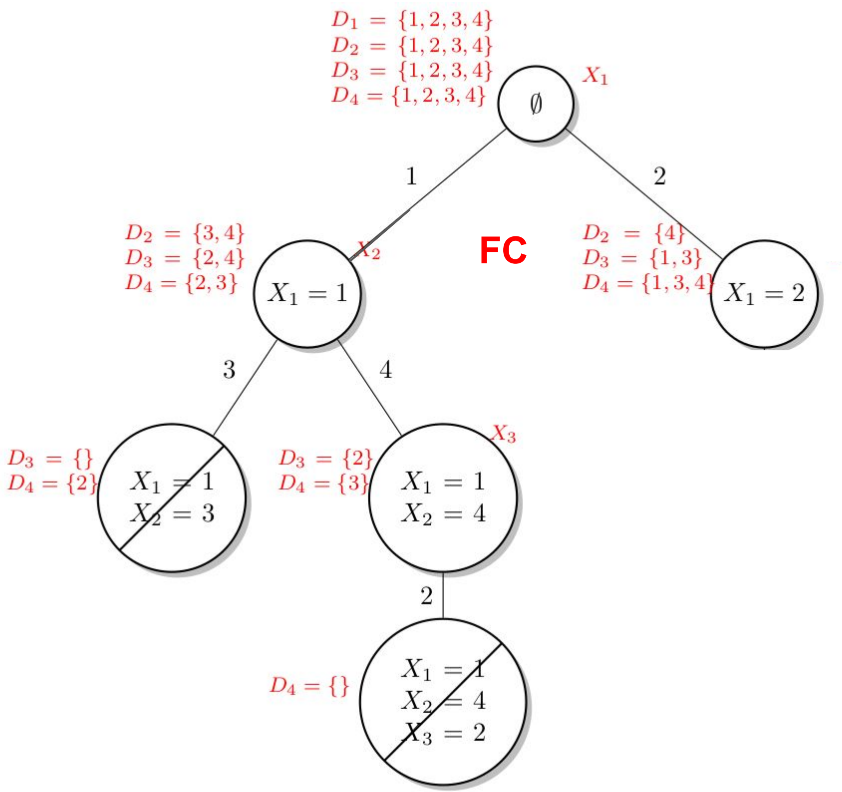


MRV: all domains have 4 values → lexicographical order

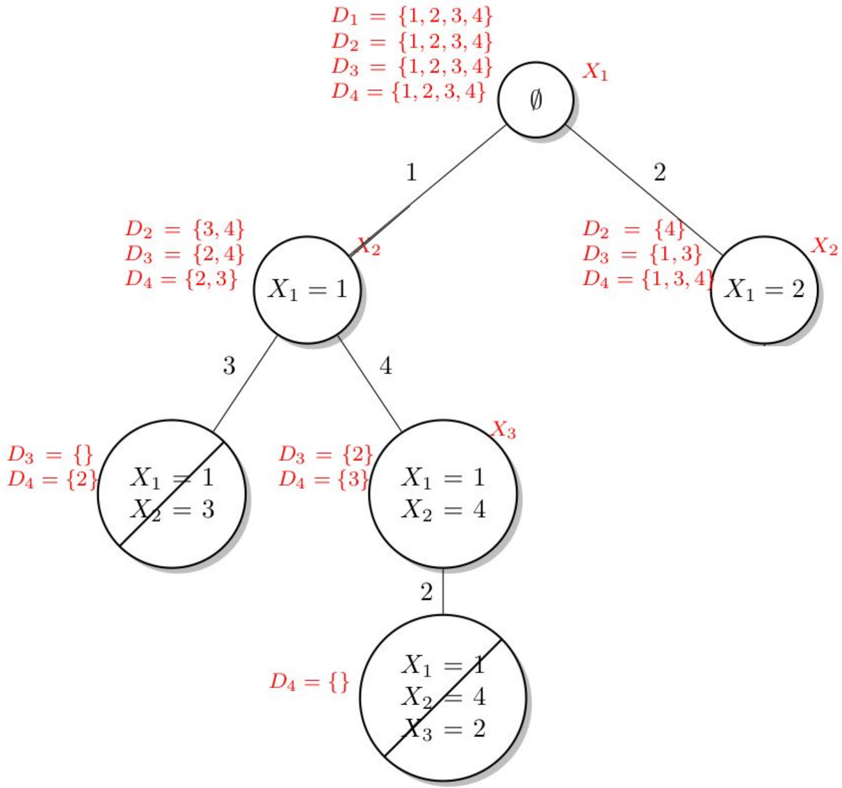


FC

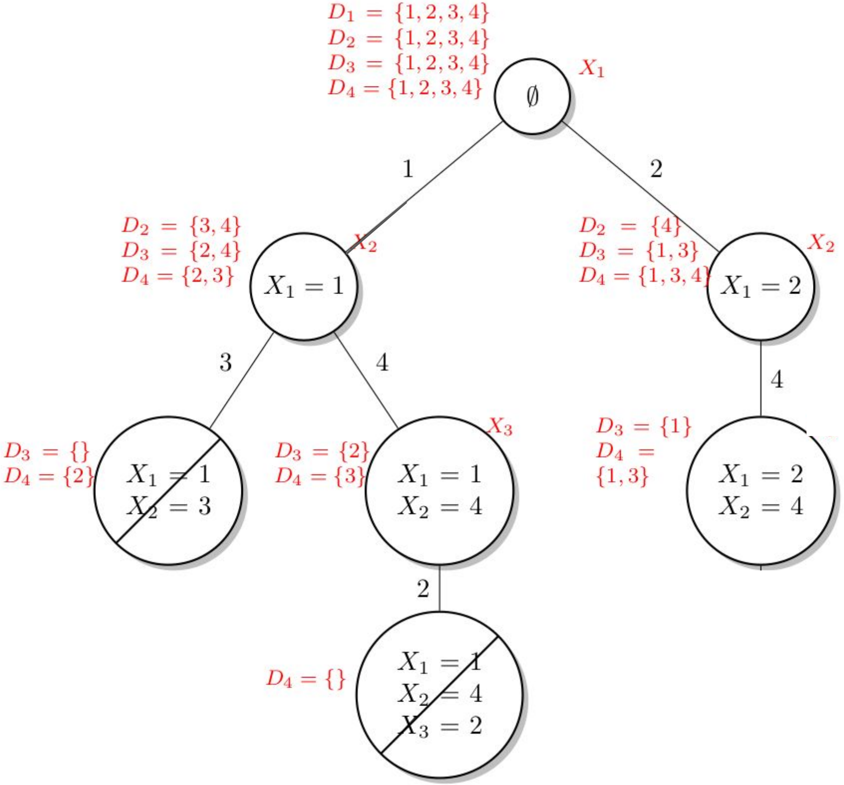
Empty domain!
$\rightarrow$ No solution



$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
FC
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
3
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
X_3
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
2
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$

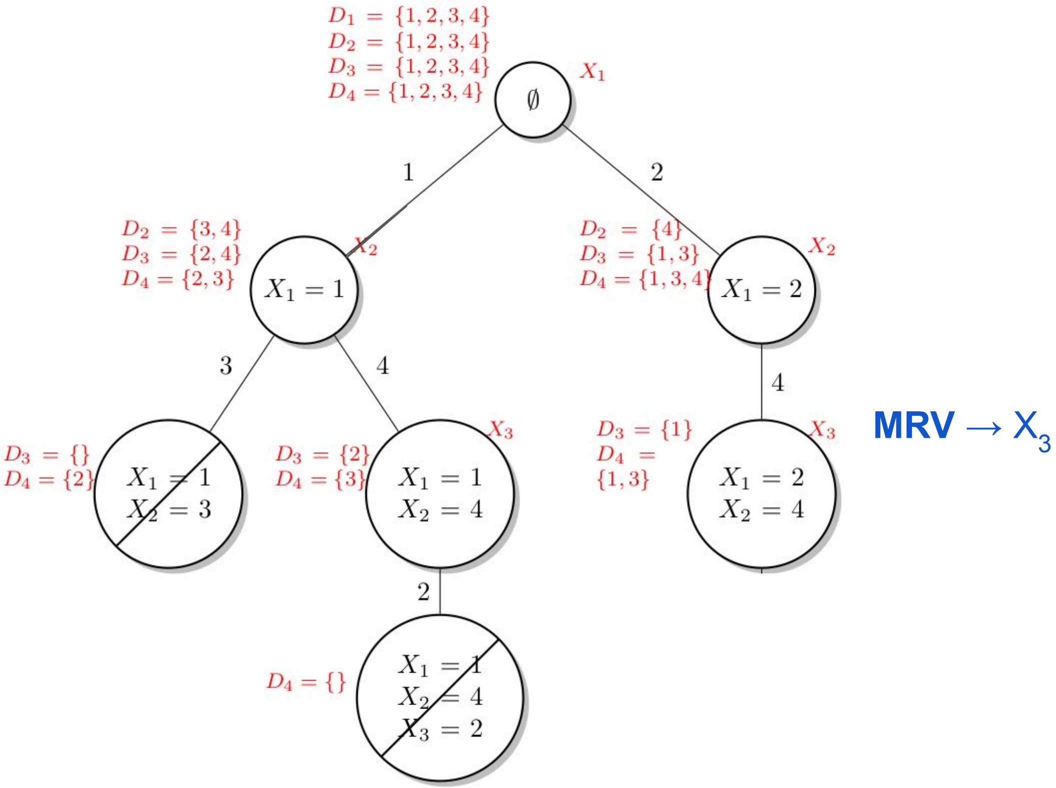


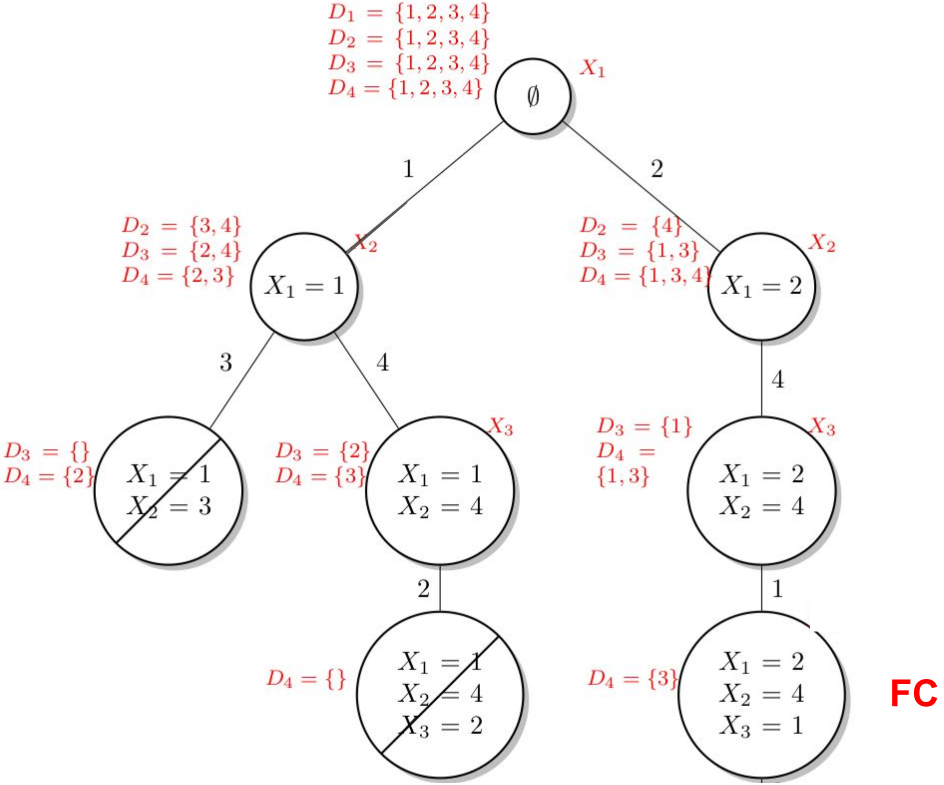
MRV $\rightarrow X_2$



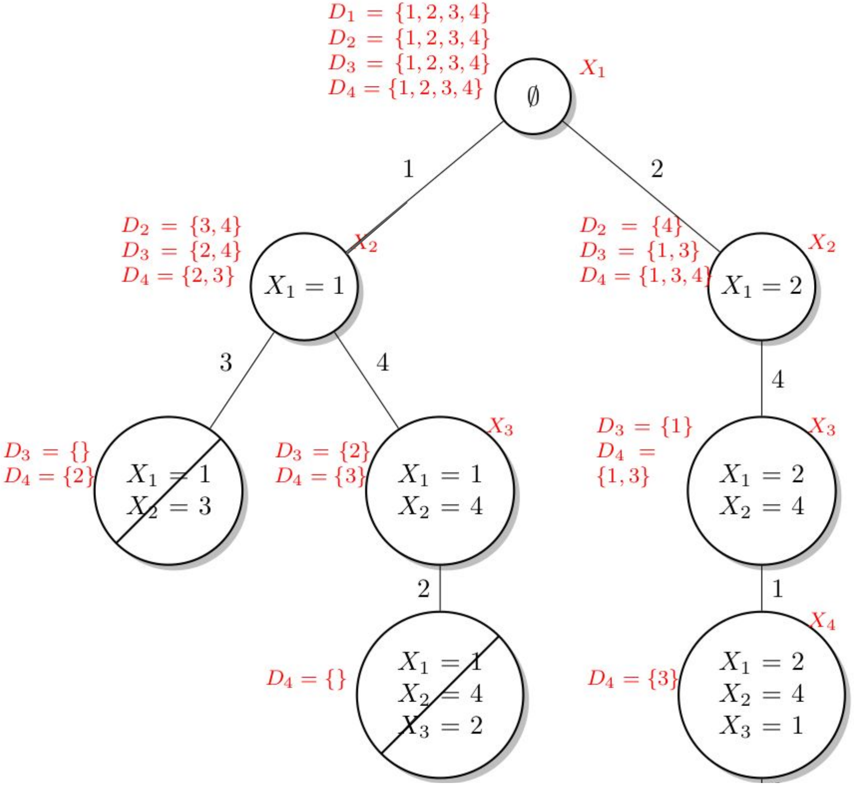
$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
X_2
3
4
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
X_3
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
2
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$

FC


$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
X_2
3
4
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
X_3
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
X_3
MRV → X_3
2
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$

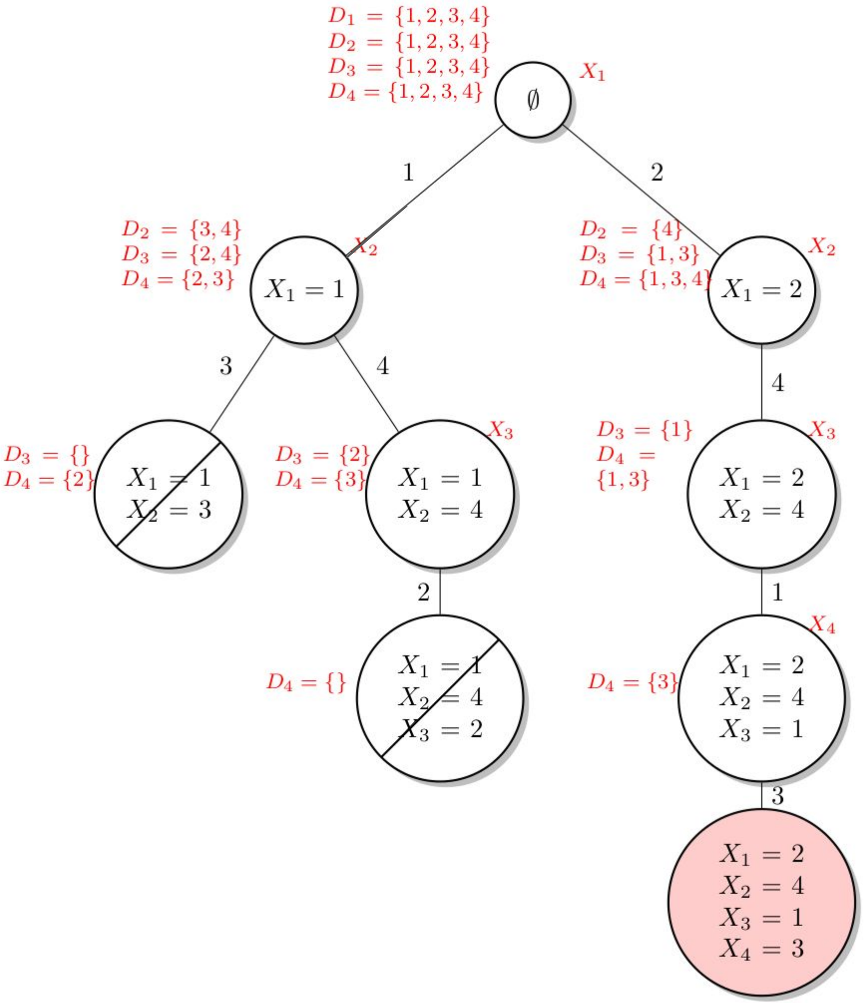


$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
X_2
$X_1 = 1$
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
X_2
$X_1 = 2$
3
4
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
X_3
$X_1 = 1$
$X_2 = 4$
$D_3 = \{1\}$
$D_4 = \{1,3\}$
X_3
$X_1 = 2$
$X_2 = 4$
2
1
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$
$D_4 = \{3\}$
$X_1 = 2$
$X_2 = 4$
$X_3 = 1$
FC



$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
X_2
3
4
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
X_3
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
X_3
2
1
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$
$D_4 = \{3\}$
$X_1 = 2$
$X_2 = 4$
$X_3 = 1$
X_4

MRV → only X_4

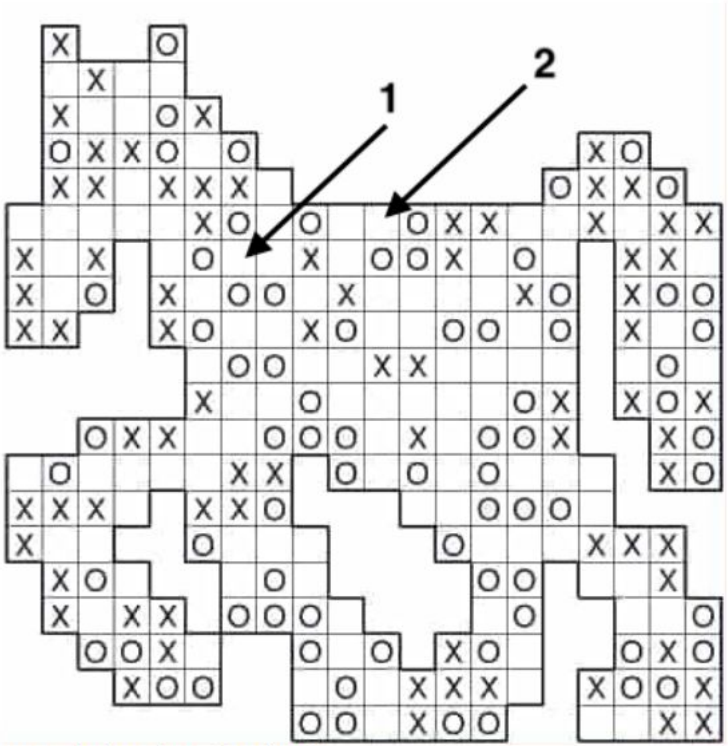

$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
2
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
X_2
3
4
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
X_3
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
X_3
2
1
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$
$D_4 = \{3\}$
$X_1 = 2$
$X_2 = 4$
$X_3 = 1$
X_4
3
$X_1 = 2$
$X_2 = 4$
$X_3 = 1$
$X_4 = 3$

Solution!

Exam - August 26, 2021

Question 2 (8 points). Constraint Satisfaction Problems (CSPs).

Consider the "never-4" puzzle, which is played on a grid (like that represented in the figure) and in which the goal is to place one of two symbols (X or O) into each cell, such that there are not any horizontal, vertical, nor diagonal sequence of 4 (or more) symbols of the same type. For instance, a vertical sequence X-X-X-X and a diagonal sequence O-O-O-O-O are illegal.



2.1 (4 points). Represent a never-4 puzzle as a CSP, reporting the set of variables, the corresponding domains, and the constraints. Be as detailed as possible when describing the constraints, specifying the variables that are involved and how they are constrained.

2.2 (2 points). Assuming that forward checking is applied, what are the possible symbols that can be inserted in the cells pointed by arrows 1 and 2 in the figure? Why? If some symbols cannot be inserted in the two cells, specify when this fact has been discovered by forward checking.

2.3 (2 points). Considering the specific CSP defined in 2.1, does the application of AC-3 algorithm further reduce the domains with respect to forward checking? Why?

Exam - August 26, 2021 - 2.1

Variables: cells of the grid.

Domains are equal for all the variables: {X, O}.

All constraints have arity 4!

(a) given any sequence of 4 horizontally-aligned cells, their values cannot be all equal;

(b) given any sequence of 4 vertically-aligned cells, their values cannot be all equal;

(c) given any sequence of 4 diagonally-aligned cells, their values cannot be all equal.

Exam - August 26, 2021 - 2.2

Cell 1: only O is left in its domain, since the X has been eliminated when the last of the three diagonal Xs has been inserted, due to one of the constraints (c).

Cell 2: both X and O are left in its domain.

Exam - August 26, 2021 - 2.3

In principle, AC-3 could further reduce the domains, but the constraints in 2.1 are **not binary**, so AC-3 cannot be applied in this case.