Arc Consistency and AC-3

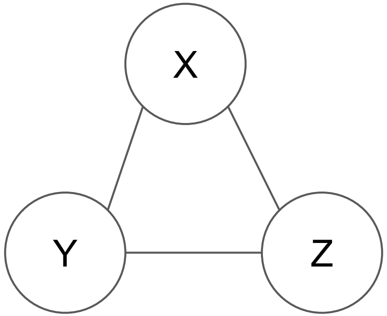
X_i is **arc consistent** (2-consistent) w.r.t. X_j if for all values $v \in D_i$ there exists $w \in D_j$ s.t. $(v,w) \in C(X_i,X_j)$.

If the CSP is arc consistent then a solution exist? Not necessarily.

If AC-3 terminates with an empty domain ⇒ **no solution exists**

If AC-3 terminates with non empty domains ⇒ **a solution might exist**

Path Consistency



3 Map coloring problem

D(X) = D(Y) = D(Z) = {red, blue}

C(X,Y) = C(X,Z) = C(Y,Z) = {(red,blue),(blue,red)}

Arc consistent but no solution exists! ⇒ Not path consistent

$\{X_i\ X_j\}$ is **path consistent** w.r.t. X_k if for all values $v \in D_i$, $w \in D_j$, and $(v,w) \in C(X_i,X_j)$ there exists $t \in D_k$ s.t. $(v,t) \in C(X_i,X_k)$ and $(w,t) \in C(X_j,X_k)$.

A path consistent CSP can still have no solution! *4 Map coloring problem*

K-consistency

A CSP is **K-consistent** if for every subset of K-1 variables,consistent assignment to those variables and for every K-th variable Y, there exists a consistent assignment for Y.

- 1-consistent = node consistent
- 2-consistent = arc consistent
- 3-consistent + all constraints are binary = path consistent

A CSP is **strongly K-consistent** if it is J-consistent for all $1 \le J \le K$.

Let N be the number of variables of a CSP. If the CSP is strongly N-consistent then a solution exists.