Foundations of Artificial Intelligence

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Exercise Session
11-24-2023

Exercise 5.3

Solve the 4-Queens problem. The problem consists of placing 4 queens on a 4x4 chess board so that no queen can attack any other. Formulate the problem as a constraint satisfaction problem and solve it using backtracking with minimum-remaining-values heuristic and forward checking. Only one solution is required.

Exercise 5.3

$X = \{x_1, x_2, x_3, x_4\}$

x_i is the row at which the queen in column i is placed

$D = \{D_1, D_2, D_3, D_4\}$ $\qquad D_1 = D_2 = D_3 = D_4 = \{1, 2, 3, 4\}$

We can express the constraints in a compact form, with $i \in \{1, 2, 3, 4\}$ and $j \in \{1, 2, 3\}$:

$$C(x_i, x_{i+j}) = \{(a, b) : a,b \in \{1, 2, 3, 4\} , |a - b| \notin \{0, j\} \}$$

Exercise 5.3

In an extensive way:

$$C(X_1, X_2) = \{\langle 1,3\rangle, \langle 1,4\rangle, \langle 2,4\rangle, \langle 3,1\rangle, \langle 4,1\rangle, \langle 4,2\rangle\}$$

$$C(X_1, X_3) = \{\langle 1,2\rangle, \langle 1,4\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,1\rangle, \langle 4,3\rangle\}$$

$$C(X_1, X_4) = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 3,1\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,2\rangle, \langle 4,3\rangle\}$$

$$C(X_2, X_3) = C(X_1, X_2)$$

$$C(X_2, X_4) = C(X_1, X_3)$$

$$C(X_3, X_4) = C(X_1, X_2)$$

Exercise 5.3

Even if not requested by the exercise, we try to apply AC-3:

$Q = \{x_1 \rightarrow x_2, x_2 \rightarrow x_1, x_1 \rightarrow x_3, x_3 \rightarrow x_1, x_1 \rightarrow x_4, x_4 \rightarrow x_1, x_2 \rightarrow x_3, x_3 \rightarrow x_2, x_2 \rightarrow x_4, x_4 \rightarrow x_2, x_3 \rightarrow x_4, x_4 \rightarrow x_3\}$

$x_1 \rightarrow x_2$: nothing

$x_2 \rightarrow x_1$: nothing

$x_1 \rightarrow x_3$: nothing

Exercise 5.3

$x_3 \rightarrow x_1$: nothing

$x_1 \rightarrow x_4$: nothing

$x_4 \rightarrow x_1$: nothing

$x_2 \rightarrow x_3$: nothing

$x_3 \rightarrow x_2$: nothing

$x_2 \rightarrow x_4$: nothing

$x_4 \rightarrow x_2$: nothing

$x_3 \rightarrow x_4$: nothing

$x_4 \rightarrow x_3$: nothing

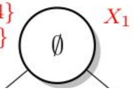
In this problem, AC-3 is unable to shrink the domains, but not all assignments of domain values is a solution!

Exercise 5.3

We apply **backtracking** with:

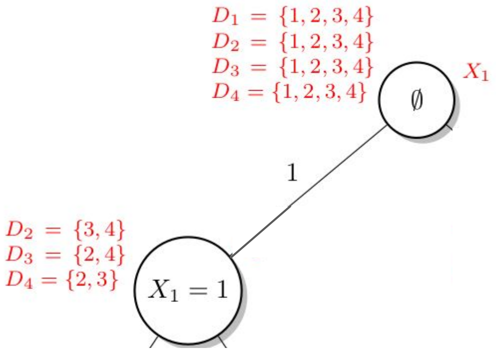
- minimum-remaining-values heuristic (**MRV**)
- forward checking (**FC**)

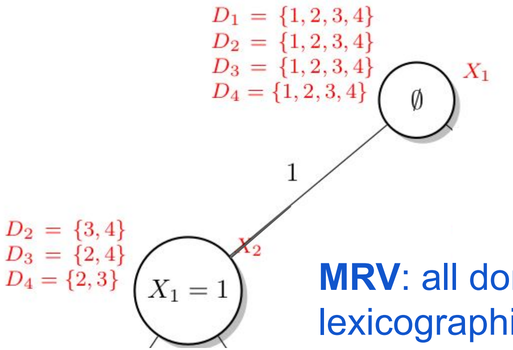
$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$



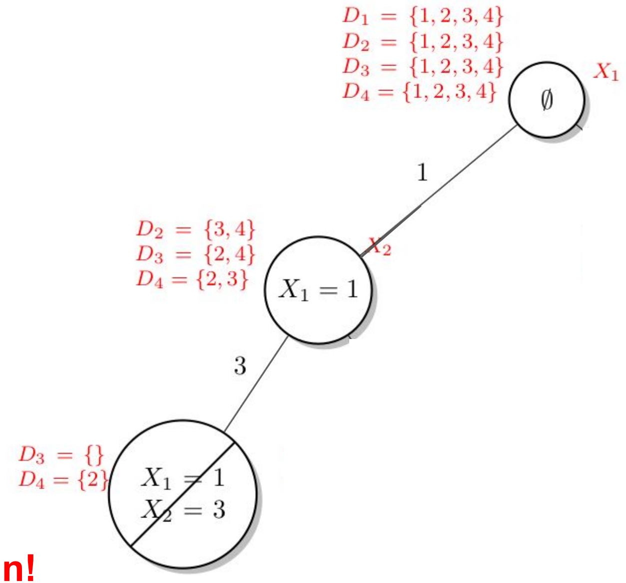
MRV: all domains have 4 values
→ lexicographical order

FC


$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$

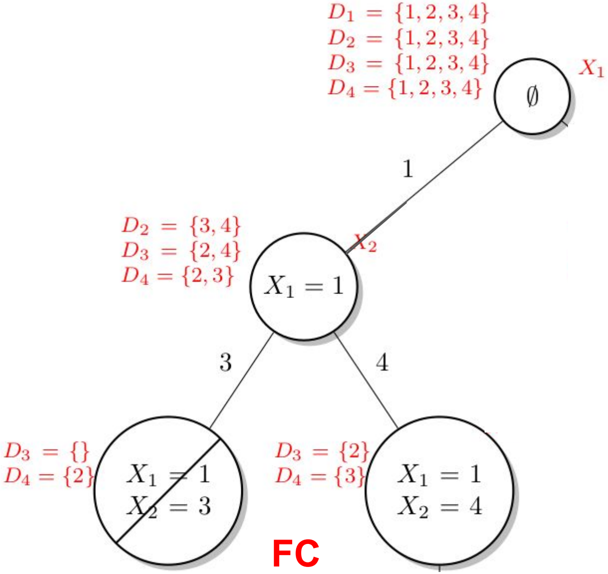


MRV: all domains have 4 values → lexicographical order

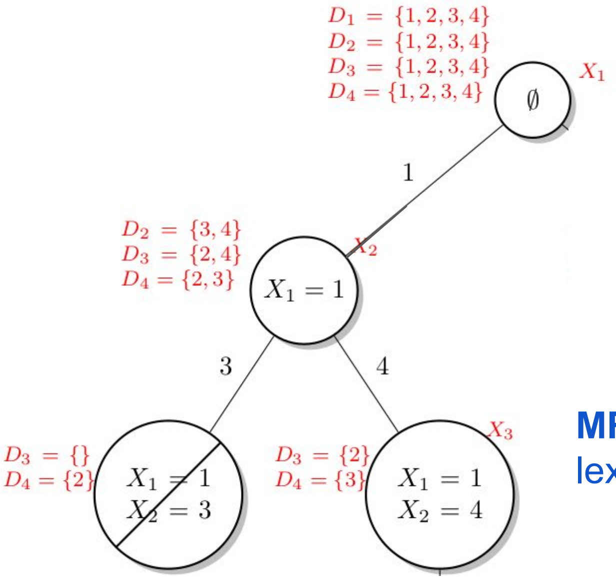


FC

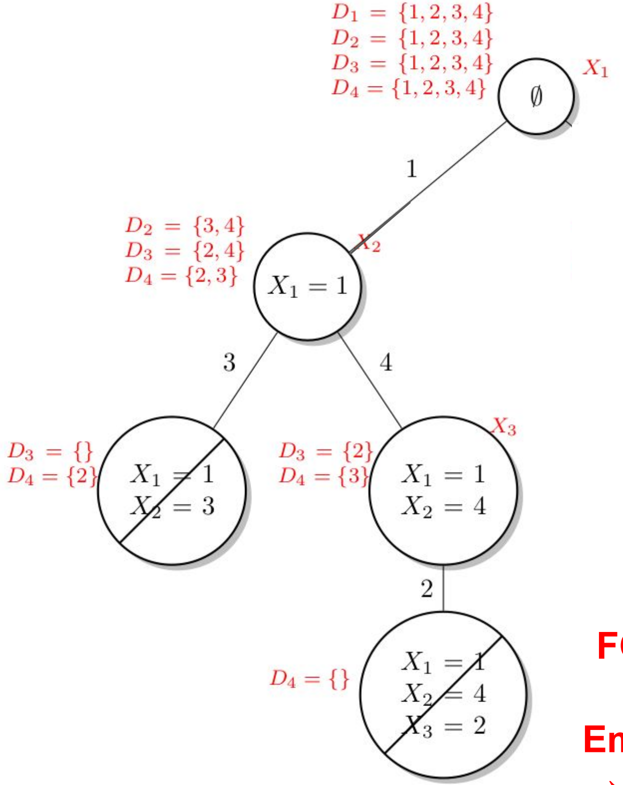
Empty domain!
→ No solution



$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
1
$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
X_2
$X_1 = 1$
3
4
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
FC

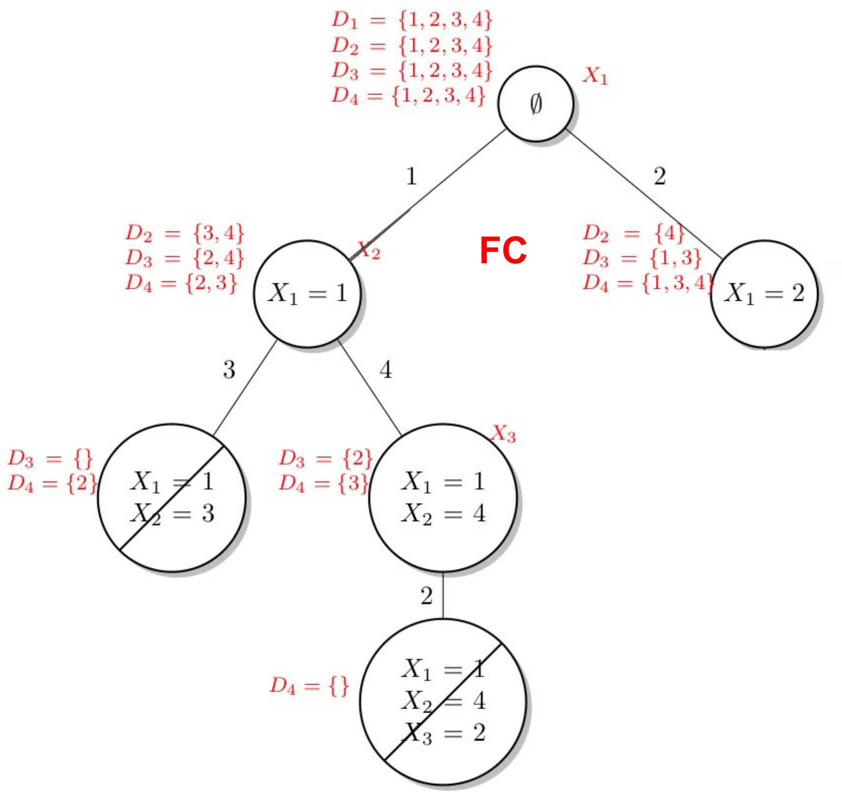


MRV: all domains have 4 values → lexicographical order



FC

Empty domain!
→ No solution



FC
$D_1 = \{1,2,3,4\}$
$D_2 = \{1,2,3,4\}$
$D_3 = \{1,2,3,4\}$
$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
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$D_2 = \{3,4\}$
$D_3 = \{2,4\}$
$D_4 = \{2,3\}$
$X_1 = 1$
X_2
$D_2 = \{4\}$
$D_3 = \{1,3\}$
$D_4 = \{1,3,4\}$
$X_1 = 2$
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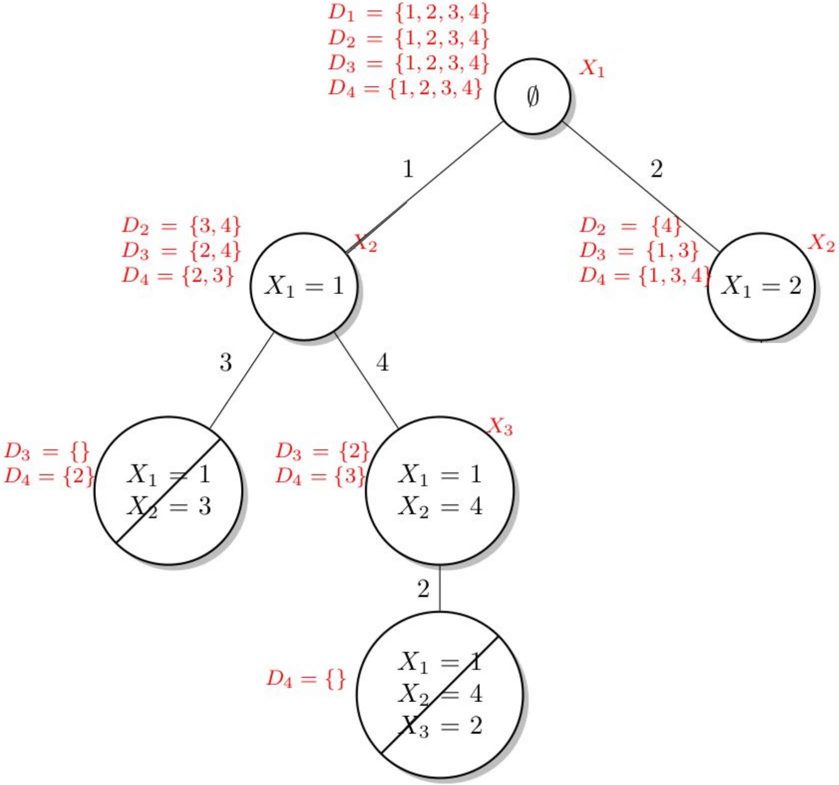
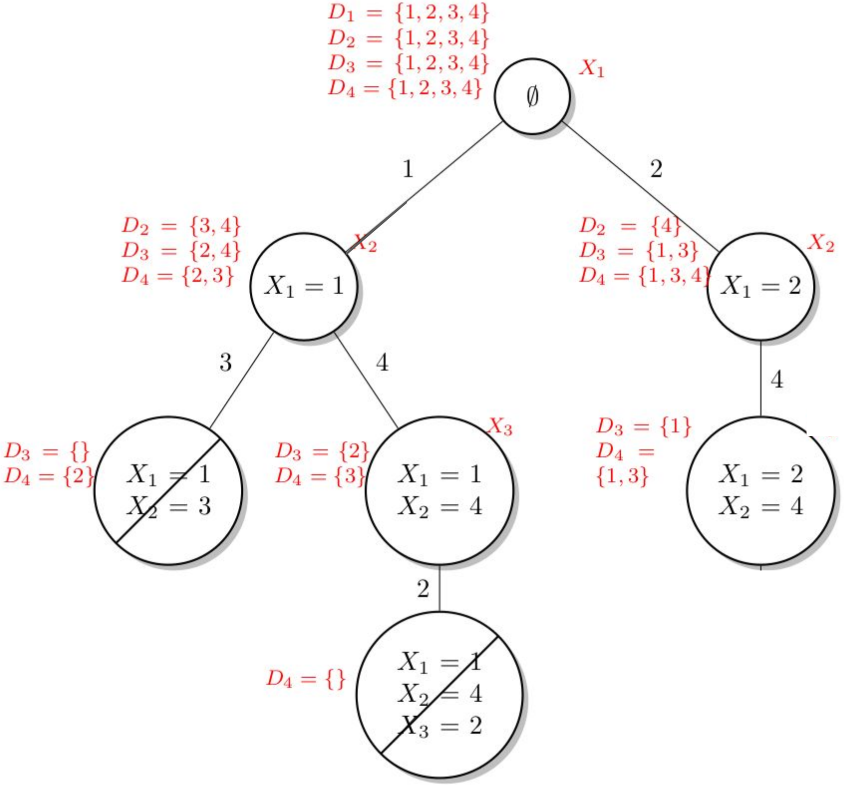
$D_3 = \{\}$
$D_4 = \{2\}$
$X_1 = 1$
$X_2 = 3$
$D_3 = \{2\}$
$D_4 = \{3\}$
$X_1 = 1$
$X_2 = 4$
X_3
2
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$



MRV $\rightarrow X_2$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$D_4 = \{1,2,3,4\}$
$\emptyset$
X_1
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$D_3 = \{2,4\}$
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$X_1 = 1$
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$D_2 = \{4\}$
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X_2
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X_3
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
2
$D_4 = \{\}$
$X_1 = 1$
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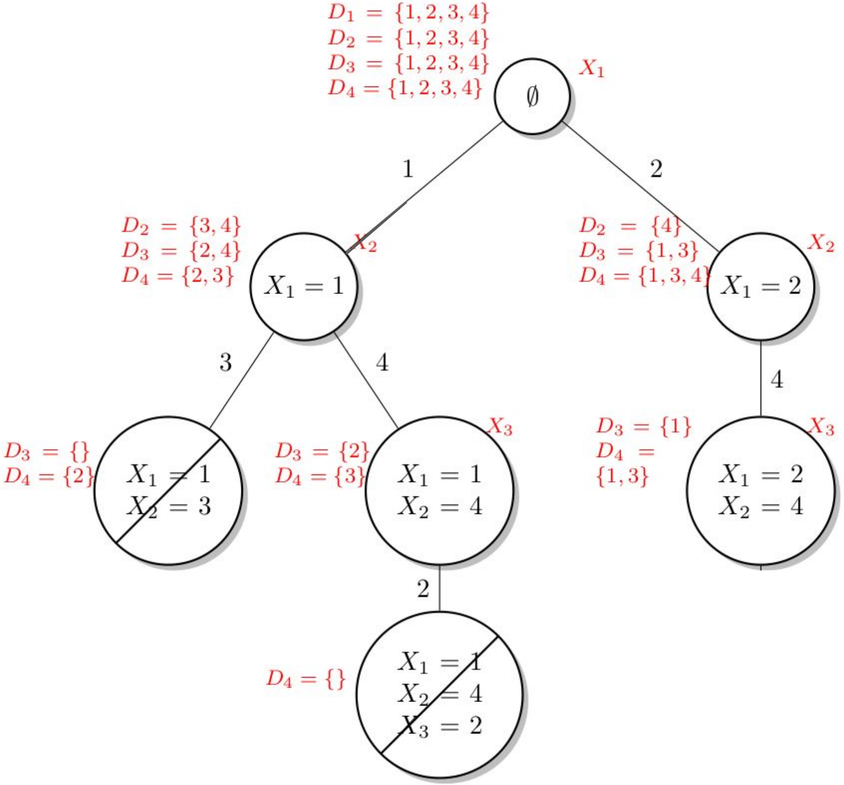
$X_3 = 2$

FC



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$D_2 = \{4\}$
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$D_3 = \{\}$
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$X_1 = 1$
$X_2 = 4$
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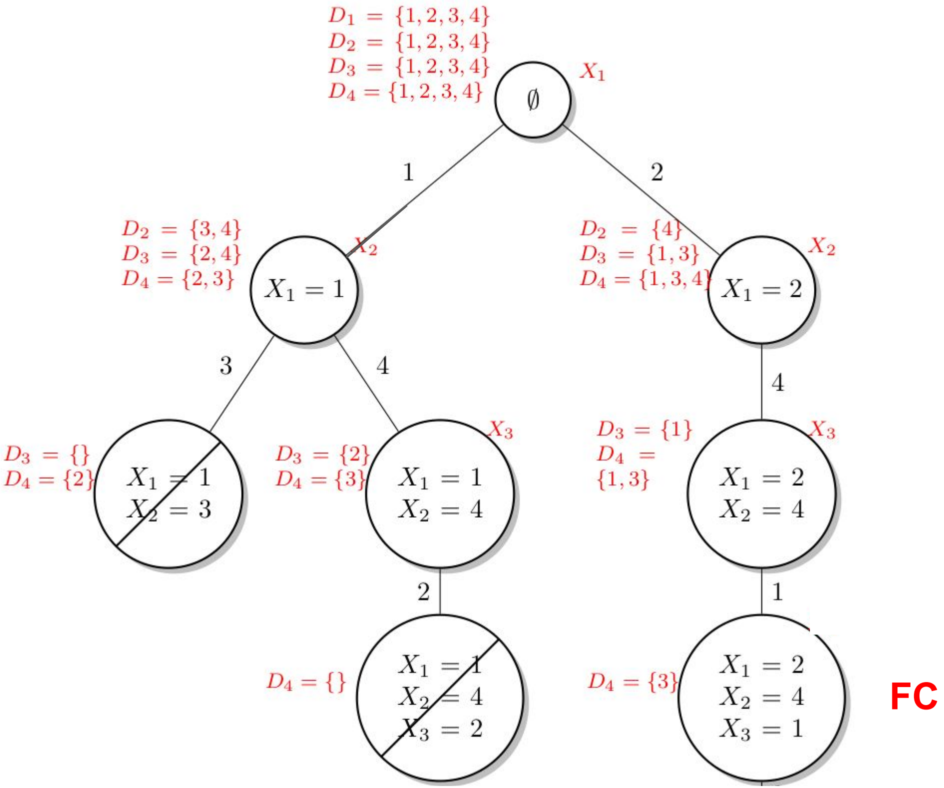
$D_3 = \{1\}$
$D_4 = \{1,3\}$
$X_1 = 2$
$X_2 = 4$
X_3
2
$D_4 = \{\}$
$X_1 = 1$
$X_2 = 4$
$X_3 = 2$

MRV → X_3



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$\emptyset$
X_1
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$D_2 = \{3,4\}$
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$X_1 = 2$
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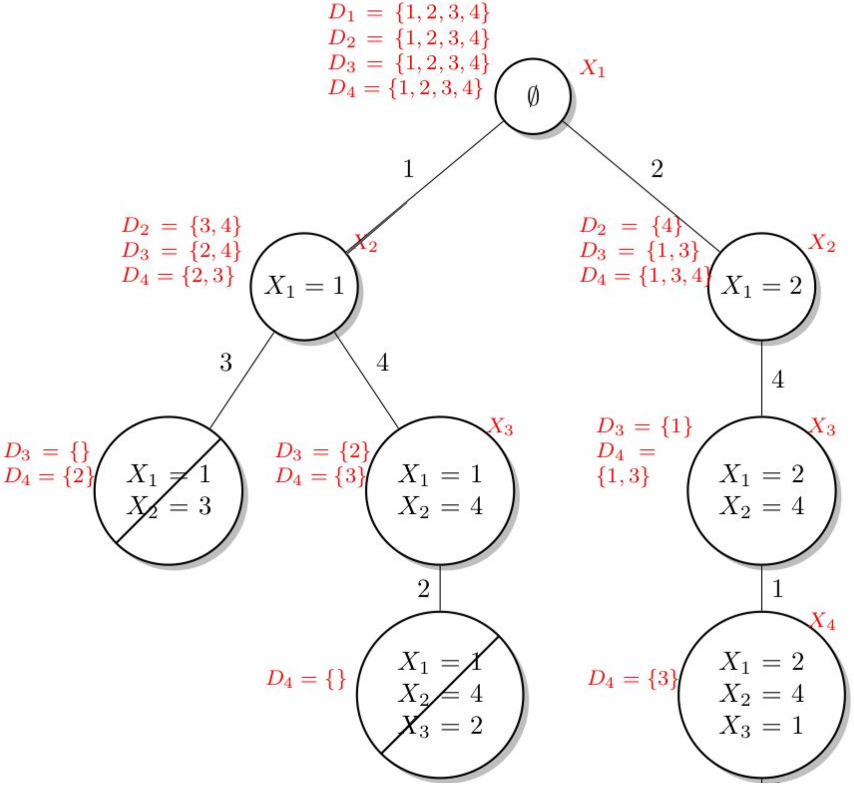
FC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$D_3 = \{1\}$
$D_4 = \{1,3\}$
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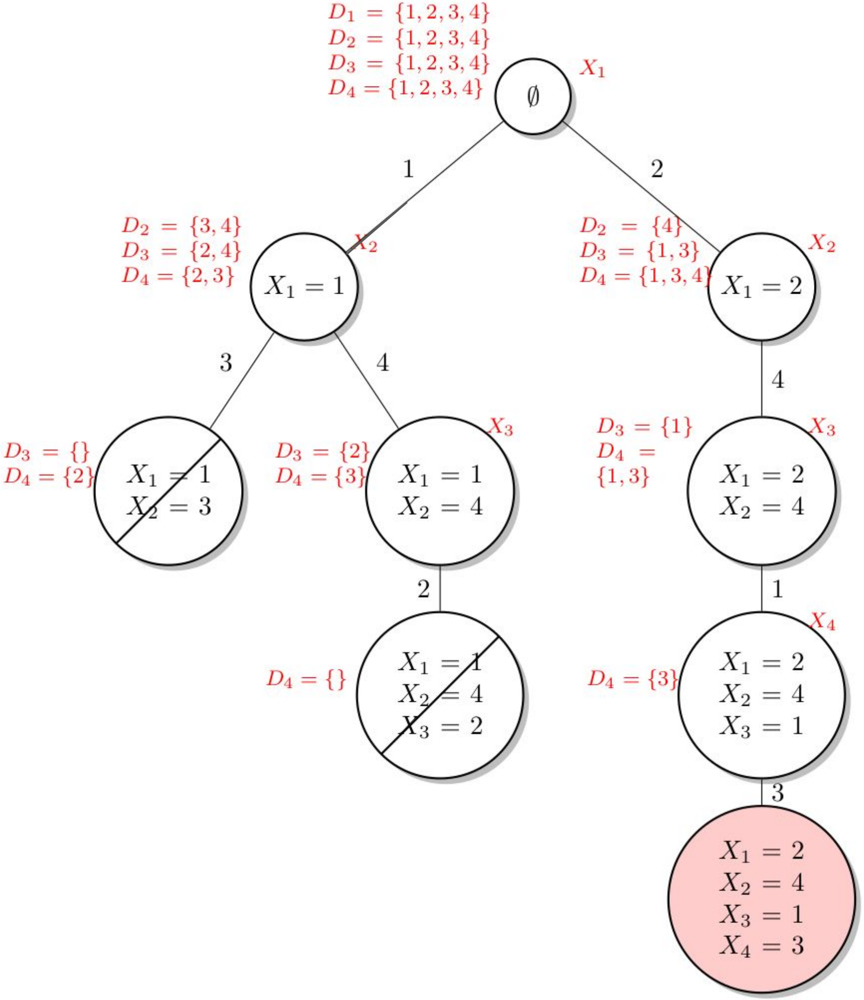
$D_4 = \{\}$
$X_1 = 1$, $X_2 = 4$, $X_3 = 2$
$D_4 = \{3\}$
$X_1 = 2$, $X_2 = 4$, $X_3 = 1$
X_4

MRV → only X_4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X_4
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$X_1 = 2$
$X_2 = 4$
$X_3 = 1$
$X_4 = 3$

Solution!

Propositional Logic (8 points). Consider the following Knowledge Base (KB) in propositional logic:

$$P$$
$$(P \wedge Q) \rightarrow R$$
$$(S \vee T) \rightarrow Q$$
$$T$$

Question 1. Apply the resolution inference algorithm (*using the unit resolution strategy*) to establish whether R is entailed by the KB.
Report all the steps.

Question 2. According to what you found in Question 1, is R entailed by the KB? Why?

Question 3. Is the unit resolution strategy complete in general? Why?