Hanoi Tower with n disks in STRIPS

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Consider the Hanoi tower problem. This puzzle has n discs, $D_1, \ldots, D_n$, with holes in their centers, and three pegs, A, B, C, on which the discs can be placed. Disc D_{i+1} is larger than disc D_i. Initially, all the discs are on peg A, with D_n on the bottom and D_1 on top. We want them on peg C in the same configuration.

The following rules apply:

- only the top disc on a peg can be moved;
- disc cannot be placed on top of a smaller one.

Model the problem in STRPS.

Solution

- Constants: A, B, C, D_1, ..., D_n
- Conditions:
  - $\texttt{Clear}(x)$: x (either peg or disk) has no disk above it [time-dependent];
  - $\texttt{On}(x,y)$: disk x is on top of y (either peg or disk) [time-dependent];
  - $\texttt{CanBePlaced}(x,y)$: disk x can be placed on top of y (either peg or disk) [time-independent].
- Initial state:

$$\begin{aligned}\texttt{Init}\Bigg(& \bigwedge_{i\in\{1,\ldots,n\},y\in\{A,B,C\}} \texttt{CanBePlaced}(D_i, y) \\ & \wedge \bigwedge_{i\in\{1,\ldots,n-1\},j\in\{i+1,\ldots,n\}} \texttt{CanBePlaced}(D_i, D_j) \\ & \wedge \texttt{On}(D_n, A) \wedge \bigwedge_{i\in\{1,\ldots,n-1\}} \texttt{On}(D_i, D_{i+1}) \\ & \wedge \texttt{Clear}(D_1) \wedge \texttt{Clear}(B) \wedge \texttt{Clear}(C)\Bigg)\end{aligned}$$

- Goal condition:[1]

$$\texttt{Goal}\left(\texttt{On}(D_n, C) \wedge \bigwedge_{i\in\{1,\ldots,n-1\}} \texttt{On}(D_i, D_{i+1}) \wedge \texttt{Clear}(D_1) \wedge \texttt{Clear}(A) \wedge \texttt{Clear}(B)\right)$$

- Action schemas: $\texttt{Move}(x, y, z)$ disk x currently on top of y (either peg or disk) is moved on top of z (either peg or disk)

$$\begin{aligned}\texttt{Action}\Big(&\texttt{Move}(x, y, z)\\ &\texttt{Precond}: \ \texttt{Clear}(x) \wedge \texttt{Clear}(z) \wedge \texttt{On}(x, y) \wedge \texttt{CanBePlaced}(x, z)\\ &\texttt{Effect}: \ \neg\texttt{Clear}(z) \wedge \neg\texttt{On}(x, y) \wedge \texttt{Clear}(y) \wedge \texttt{On}(x, z)\Big)\end{aligned}$$

[1] Given the next action schema, we can just use as goal condition: $\texttt{Goal}(\texttt{Clear}(A) \wedge \texttt{Clear}(B))$.