Compatible Reward Inverse Reinforcement Learning

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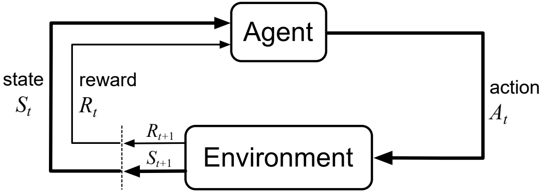
Reinforcement Learning

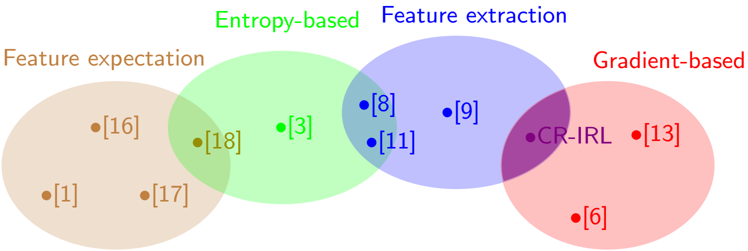
- *Reinforcement Learning* (RL) [14]:
    - learning by interaction.
- Parametric policy: $\pi_{\boldsymbol{\theta}}$.
- Expected return: $J(\boldsymbol{\theta}) = \underset{\tau \sim p_{\boldsymbol{\theta}}}{\mathbb{E}} \left[\sum_{t=0}^{T(\tau)} \gamma^t R(s_{\tau,t}, a_{\tau,t}) \right]$.



State of the Art

- RL requires a *reward function* $R(s, a)$.
- Designing a suitable reward function is challenging (e.g., car driving task [1]).
- Learning from demonstrations (*Imitation Learning*):
  - Behavioral Cloning (BC) [2];
  - Inverse Reinforcement Learning (IRL) [12].

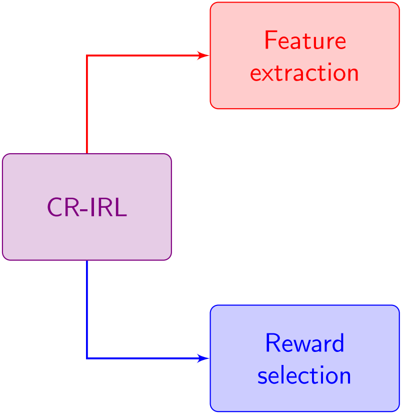
Motivations and Goals



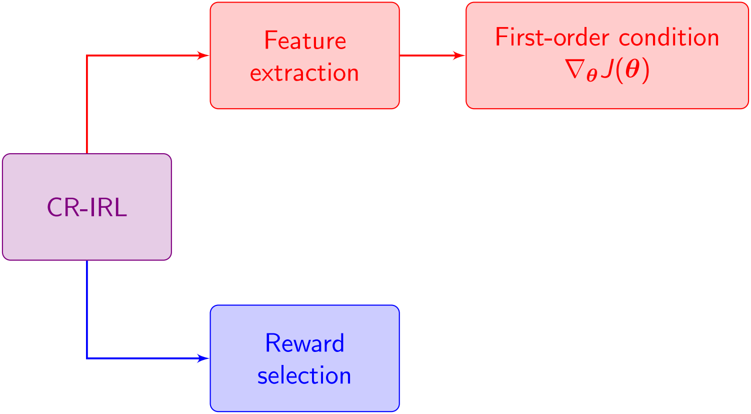
- **Motivations** – state-of-the-art IRL algorithms require:
  - the environment transition model;
  - a set of engineered reward features.
- **Goal** – design an IRL algorithm requiring only expert's trajectories.

Compatible Reward Inverse Reinforcement Learning

Overview



Compatible Reward Inverse Reinforcement Learning
Overview



Compatible Reward Inverse Reinforcement Learning

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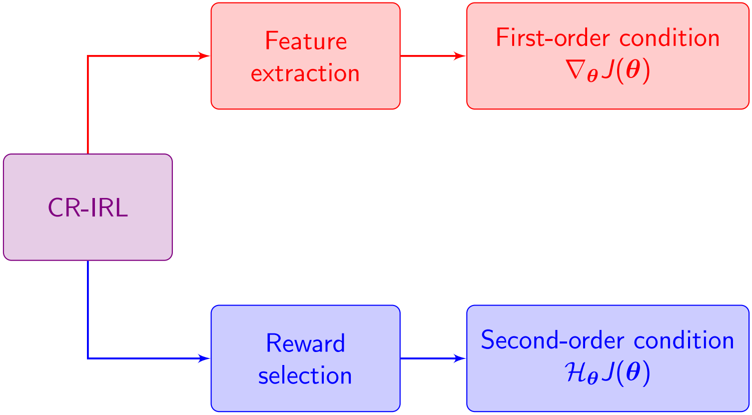

Feature extraction

Expert's COmpatible Value Features

- Extract the *compatible* value functions.
- First-order condition on *policy gradient* [15]:

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \int_{\mathcal{S}} \int_{\mathcal{A}} \delta_{\mu,\gamma}^{\pi_{\boldsymbol{\theta}}}(s,a) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(a|s) Q^{\pi_{\boldsymbol{\theta}}}(s,a) \mathrm{d}s \mathrm{d}a = \mathbf{0}.$$

- *Expert's COmpatible Value Features* (ECO-Q):

- Extension to unvisited state-action pairs with KNN.

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$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \mathbf{0} \longrightarrow \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}} \perp Q^{\pi_{\boldsymbol{\theta}}} \longrightarrow \boldsymbol{\Phi} = \mathrm{null}\left(\nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}{}^T \mathbf{D}_{\mu,\gamma}^{\pi_{\boldsymbol{\theta}}}\right)$$

- Extension to unvisited state-action pairs with KNN.

Feature extraction

Expert's COmpatible Reward Features

- Extract the *compatible* reward functions.
- *Expert's COmpatible Reward Features* (ECO-R):
  - model-based ECO-R

$$\mathbf{\Psi}^{MB} = (\mathbf{I} - \gamma \mathbf{P} \boldsymbol{\pi}_{\boldsymbol{\theta}})\mathbf{\Phi};$$

  - model-free ECO-R

$$\mathbf{\Psi}^{MF} = \left(\mathbf{I} - \tilde{\boldsymbol{\pi}}_{\boldsymbol{\theta}}\right)\mathbf{\Phi}.$$

Reward selection

Preliminaries

- Linear reward parametrization:

$$\mathbf{r} = \mathbf{\Psi}\boldsymbol{\omega}.$$

- Select a reward function that:
  - is maximum of $J(\boldsymbol{\theta})$;
  - penalizes maximally deviations from the expert's policy.
- Second-order conditions on *policy Hessian* [10].

Reward selection

Second-Order criteria

- Second-order optimality criteria:
  - minimize the *maximum eigenvalue*;
  - minimize the *trace*.
- Negative semidefinite Hessian as constraint.
- Second-order *trace heuristic*:
  - reduced space of ECO-R;
  - closed-form solution:

$$\omega = \frac{\mathbf{tr}}{\|\mathbf{tr}\|_2}.$$

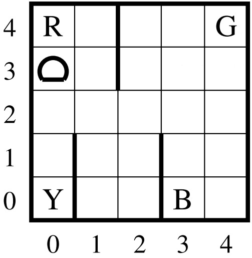
Experiments

- Environments:
  - **Taxi problem** [4] (finite);
  - Linear-Quadratic Gaussian Regulator [5] (continuous);
  - **Car on the Hill** [7] (continuous).
- Metrics:
  - **Learning speed**;
  - Average return;
  - Parameter distance;
  - Policy distance (KL-divergence).

Taxi

Preliminaries

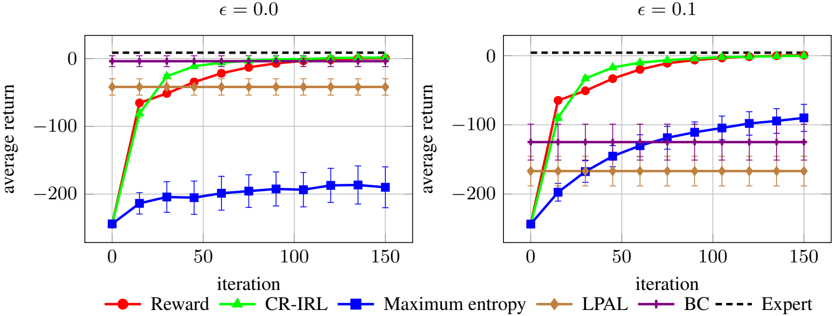
- Finite episodic problem.
- Expert's ϵ-Boltzmann policy with state-dependent features.



Taxi

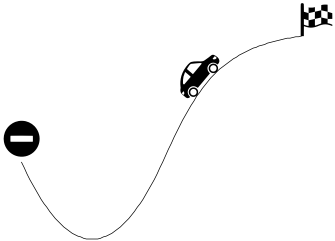
Learning speed

- Comparison with Maximum Entropy IRL [18], LPAL [16] and BC.



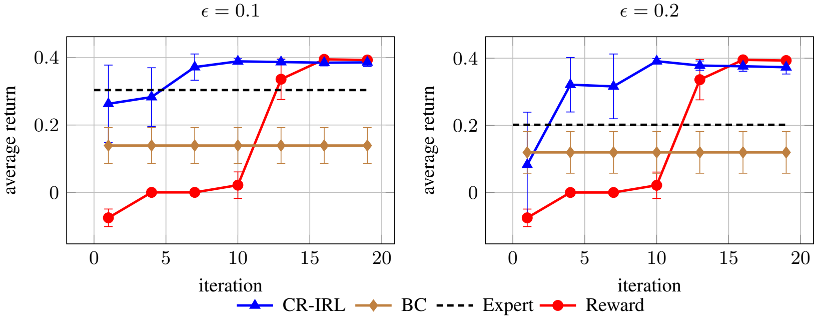
Car on the Hill
Preliminaries

- Continuous episodic problem.
- Expert's policy computed via FQI [7].



Car on the Hill

Learning speed



Conclusions

- Contributions
  - Construction of both features and reward function.
  - Faster learning speed w.r.t. the original reward function.
  - Better performance w.r.t. BC and several IRL methods.
- Paper submitted to NIPS.
- Future Works
  - Theoretical analysis of the maximum likelihood policy.
  - Direct construction of ECO-R.

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Policy Rank

- How large is the space of ECO-Qs?

Definition

Let $\pi_{\boldsymbol{\theta}}$ a policy with k parameters belonging to the class Π_Θ and differentiable in $\boldsymbol{\theta}$. The policy rank is the dimension of the space of the linear combinations of the partial derivatives of $\pi_{\boldsymbol{\theta}}$ w.r.t. $\boldsymbol{\theta}$:

$$\mathrm{rank}(\pi_{\boldsymbol{\theta}}) = \dim(\Gamma_{\pi_{\boldsymbol{\theta}}}), \quad \Gamma_{\pi_{\boldsymbol{\theta}}} = \{\nabla_{\boldsymbol{\theta}}\pi_{\boldsymbol{\theta}}\boldsymbol{\alpha} : \boldsymbol{\alpha} \in \mathbb{R}^k\}.$$

- The policy rank quantifies how much a policy is *informative* for recovering the optimal value function (and so the optimal reward function).
- In finite domains it holds: $\mathrm{rank}(\pi_{\boldsymbol{\theta}}) \leq \min\{k, |\mathcal{S}||\mathcal{A}| - |\mathcal{S}|\}$.

Reward shaping

- Given a reward function R inducing an optimal policy π, which is the class of rewards preserving the optimality of π?
- Optimality of π is preserved for *potential-based* shaping functions:

$$R'(s,a) = R(s,a) + \gamma \int_{\mathcal{S}} P(s'|s,a)\chi(s')\mathrm{d}s' - \chi(s)$$

- A smart choice is $\chi(s) = V^{\pi}(s)$, so we get the *advantage function*:

$$R'(s,a) = Q^{\pi}(s,a) - V^{\pi}(s,a) = A^{\pi}(s,a).$$

- The advantage function allows running RL algorithms with smaller γ.

Multi-objective second-order criteria

- Ideally we would like to minimize "all" the eigenvalues of the policy Hessian.

$$\begin{aligned} \underset{\boldsymbol{\omega}\in\mathbb{R}^p}{\text{minimize}} \quad & \boldsymbol{\lambda}(\boldsymbol{\omega}) = \left(\lambda_1(\boldsymbol{\omega}), \lambda_2(\boldsymbol{\omega}), ..., \lambda_k(\boldsymbol{\omega})\right) \\ \text{subject to} \quad & \mathcal{H}_{\boldsymbol{\theta}} J(\boldsymbol{\theta}, \boldsymbol{\omega}) + \epsilon \mathbf{I} \preceq 0. \end{aligned}$$

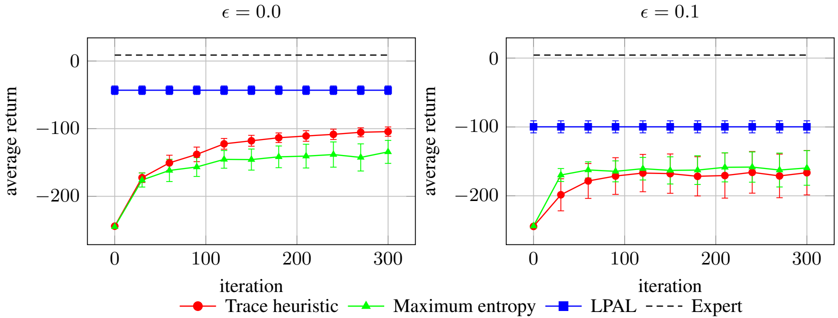
- We consider linear scalarizations:

$$L(\boldsymbol{\lambda}(\boldsymbol{\omega}), \gamma) = \sum_{i=1}^{k} \gamma_i \lambda_i(\boldsymbol{\omega}) = \boldsymbol{\gamma}^T \boldsymbol{\lambda}(\boldsymbol{\omega}).$$

- $\gamma_1 = 1$ and $\gamma_i = 0$ for $i = 2, 3, ..., k$ we get *maximum eigenvalue* optimality criterion.
- $\gamma_i = 1$ for $i = 1, 2, ..., k$ we get *trace* optimality criterion.

Taxi

Comparison with PVF



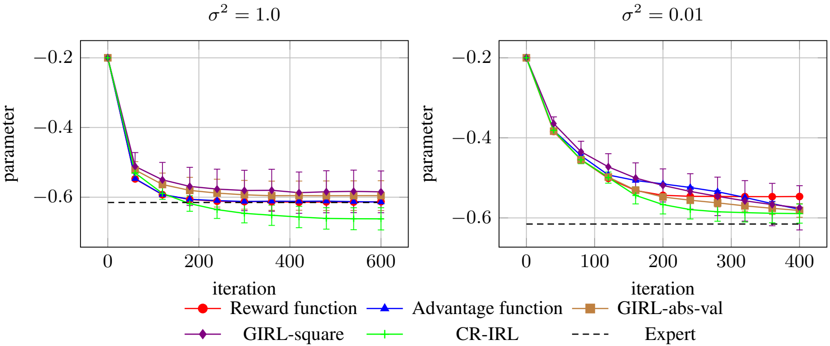
Linear Quadratic Gaussian Regulator (LQG)

Preliminaries

- Continuous infinite-horizon problem.
- Gaussian (noisy) expert's policy, the mean is the optimal action.
- Variance to test resilience to imperfect experts.

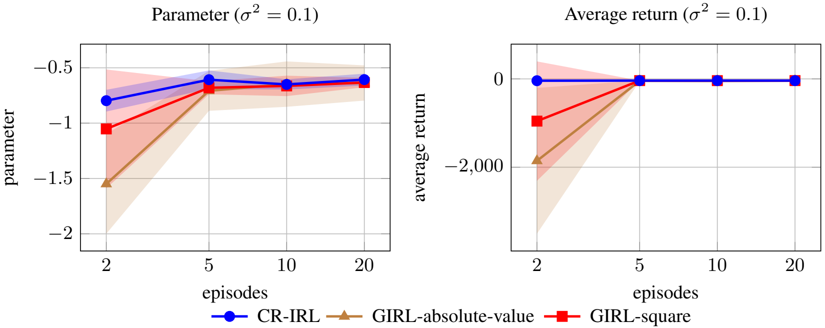
1D-LQG
Learning speed

- Comparison with GIRL.
- Train a Gaussian policy with REINFORCE.



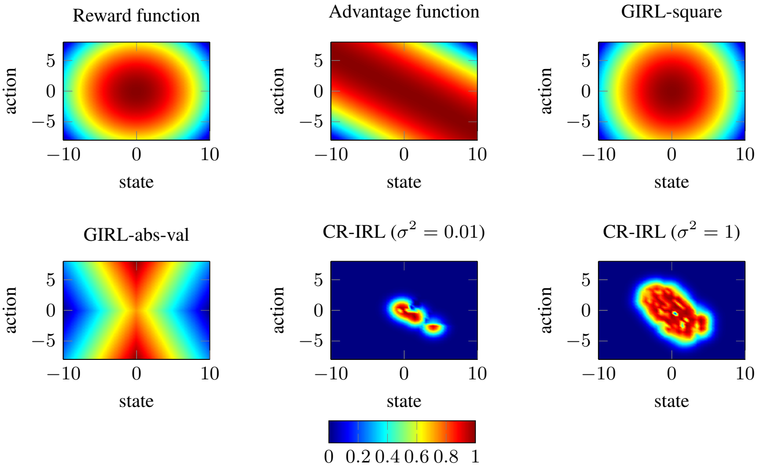
1D-LQG

Sensitivity to the number of expert's demonstrations

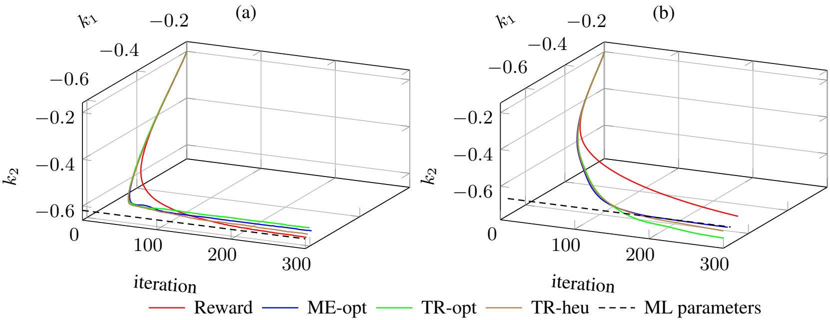


1D-LQG

Recovered rewards

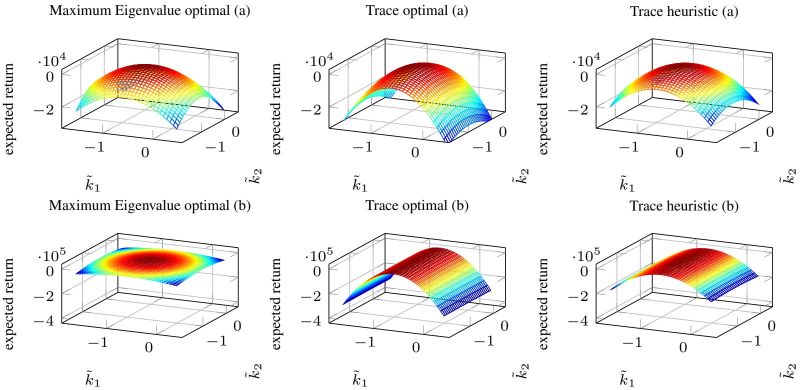


2D-LQG
Learning speed



2D-LQG

Shape of expected return



Car on the Hill

Trajectories